Westfälische Wilhelms-Universität Münster

Applied Mathematics Münster

Reduced Basis and Domain Decomposition for Maxwell's Equations

Reduced Basis Summer School 2013 - Kopp

living.knowledge WWU Münster

Andreas Buhr

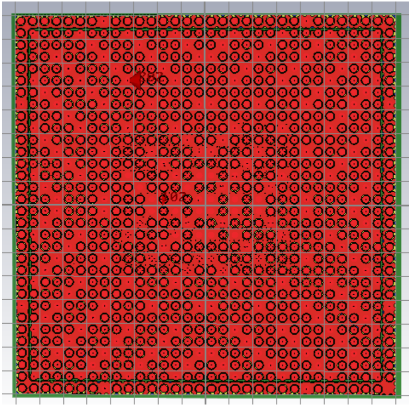
20. August 2013

Simulating a Chip Carrier in a Flip Chip Package

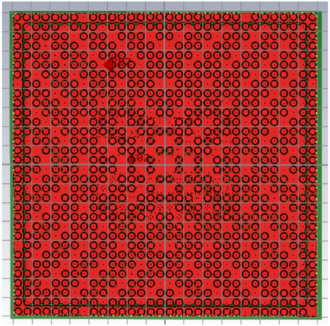
About this talk

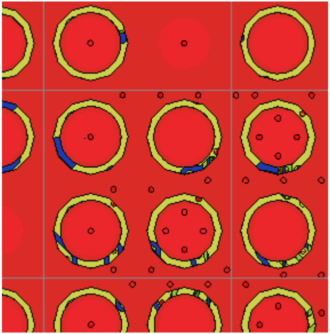
- I just started working on my PhD.
- There are no results yet, everything I show is just ideas.
- All screenshots I show are made with CST Studio Suite.
- The chip carrier shown in this presentation is from a CAD simulation tool benchmark published by IBM.

Chip Carrier, seen from below

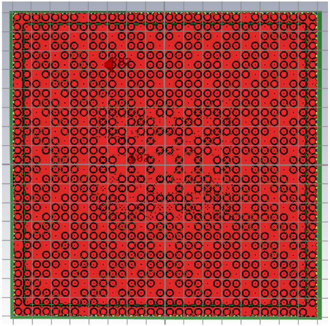


Chip Carrier, seen from below

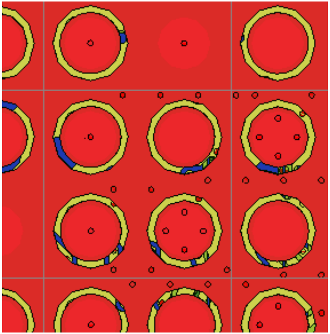




Chip Carrier, seen from below

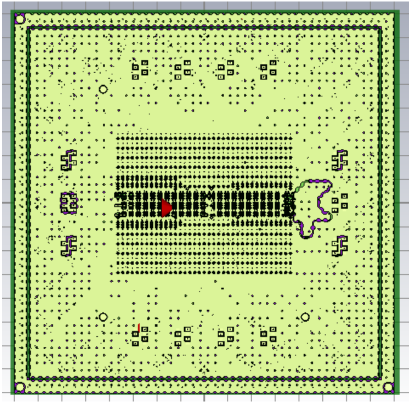


size $\approx (32mm)^2$

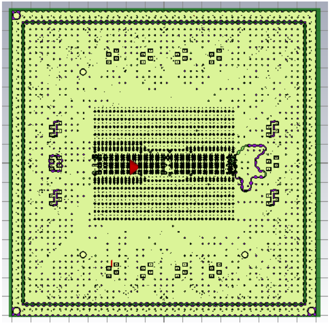


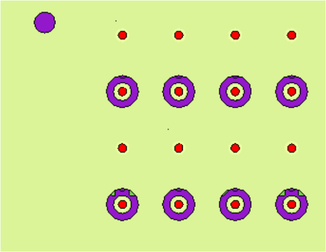
size $\approx (3.5mm)^2$

Chip Carrier, seen from above

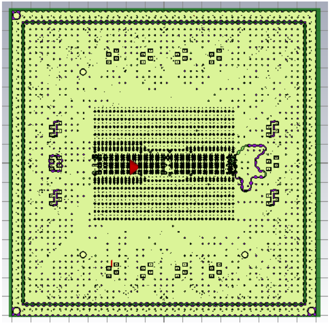


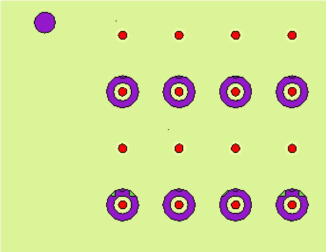
Chip Carrier, seen from above





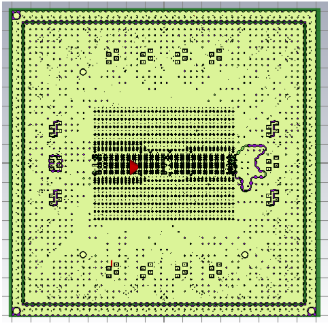
Chip Carrier, seen from above

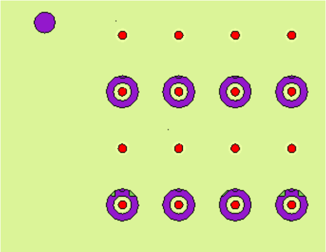




distance between two connectors
$\approx 0.2mm$

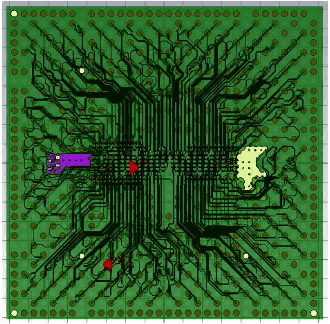
Chip Carrier, seen from above

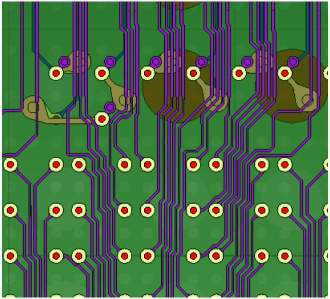




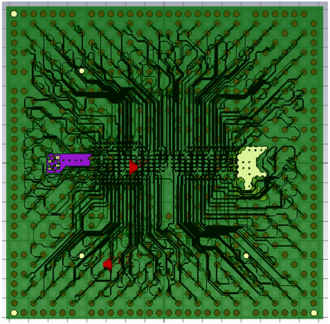
distance between two connectors
$\approx 0.2mm$
$\approx$ 3 human hairs

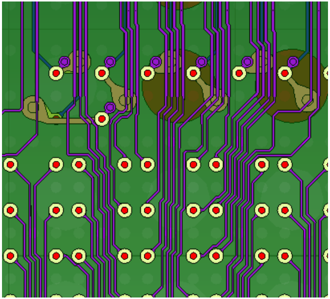
Chip Carrier, interior





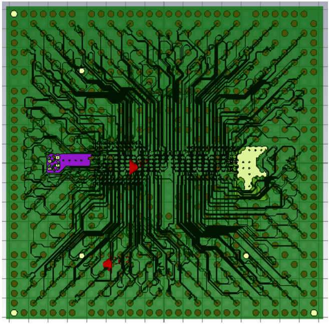
Chip Carrier, interior

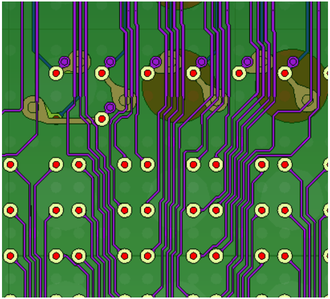




thickness of traces
$\approx 0.025mm$

Chip Carrier, interior



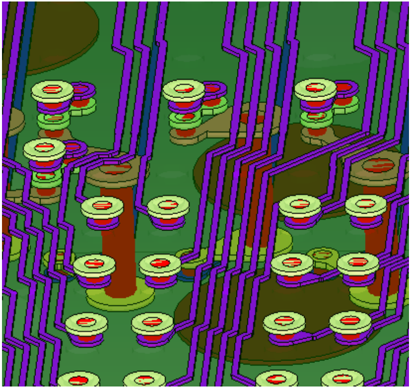


thickness of traces

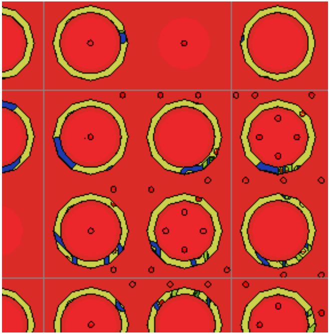
$\approx 0.025mm$

$\approx \frac{1}{3}$ human hair

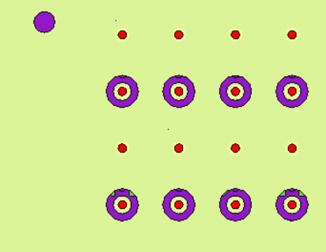
Chip Carrier, interior



Quantity of Interest: Transfer Function



from



to

Mathematical Modelling

Solution of Maxwell's Equations:

$$\nabla \times E = -\frac{\partial}{\partial t} B \tag{1}$$

$$\nabla \cdot D = \rho \tag{2}$$

$$\nabla \times H = \partial_t D + j \tag{3}$$

$$\nabla \cdot B = 0 \tag{4}$$

$$D = \epsilon E \tag{5}$$

$$B = \mu H \tag{6}$$

Mathematical Modelling

Quantities in Maxwell's Equations:

E	electric field
D	electric flux
H	magnetic field
B	magnetic flux
ρ	charge density
ϵ	electric permittivity
μ	magnetic permeability

Mathematical Modelling

$$\nabla \times \frac{1}{\mu} \nabla \times E + \frac{\partial^2}{\partial t^2} \epsilon E = -\frac{\partial}{\partial t} j \tag{7}$$

Mathematical Modelling

$$\nabla \times \frac{1}{\mu} \nabla \times E + \frac{\partial^2}{\partial t^2} \epsilon E = -\frac{\partial}{\partial t} j \quad (7)$$

With time harmonic ansatz: $E(x, t) = Re(E(x, \omega) \cdot e^{i \omega t})$

Mathematical Modelling

$$\nabla \times \frac{1}{\mu} \nabla \times E + \frac{\partial^2}{\partial t^2} \epsilon E = -\frac{\partial}{\partial t} j \tag{7}$$

With time harmonic ansatz: $E(x,t) = Re(E(x,\omega) \cdot e^{i\omega t})$

$$\nabla \times \frac{1}{\mu} \nabla \times E - \omega^2 \epsilon E = -i\omega j \tag{8}$$

Where is the Parametrization?

$$\nabla \times \frac{1}{\mu} \nabla \times E - \omega^2 \epsilon E = -i\omega j \tag{9}$$

Where is the Parametrization?

$$\nabla \times \frac{1}{\mu} \nabla \times E - \omega^2 \epsilon E = -i\omega j \tag{9}$$

- The parametrization is in ω. We always need solutions in a range, e.g. from 0 to 10 GHz:

$$\omega \in [0, 10^{10}]$$

Where is the Parametrization?

$$\nabla \times \frac{1}{\mu} \nabla \times E - \omega^2 \epsilon E = -i\omega j \qquad (9)$$

- The parametrization is in ω. We always need solutions in a range, e.g. from 0 to 10 GHz:

$$\omega \in [0, 10^{10}]$$

- Note that operator is affin separable and linear.

Where is the Parametrization?

$$\nabla \times \frac{1}{\mu}\nabla \times E - \omega^2 \epsilon E = -i\omega j \qquad (9)$$

- The parametrization is in ω. We always need solutions in a range, e.g. from 0 to 10 GHz:

$$\omega \in [0, 10^{10}]$$

- Note that operator is affin separable and linear.
- Metal is modelled as Dirichlet boundary condition:

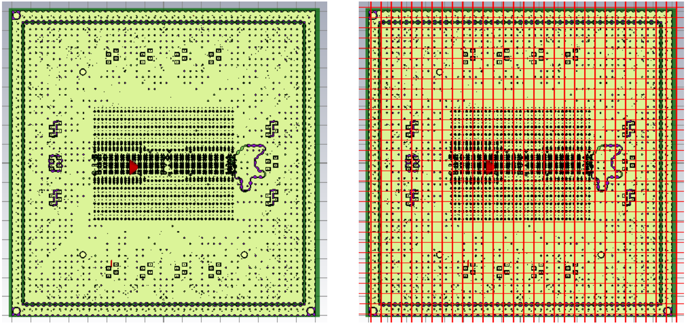
$$E \times n = 0$$

where n is the outer normal vector on the metal surface.

Main Motivation

- Chip is not simulated once, but multiple times with small changes.
- Changes often cannot be described as parametrized geometries.
- Changes are mostly local.
- Idea: Create localized basis. After local change, only regenerate basis in changed region.

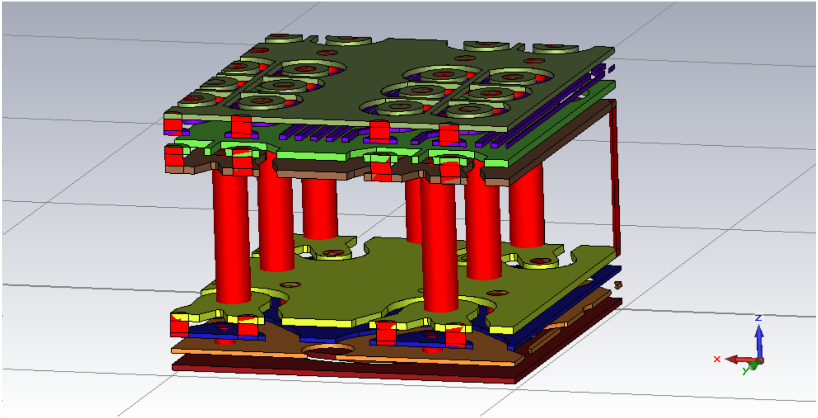
Domain Decomposition



Creating quite small domains, but much larger than geometric details.

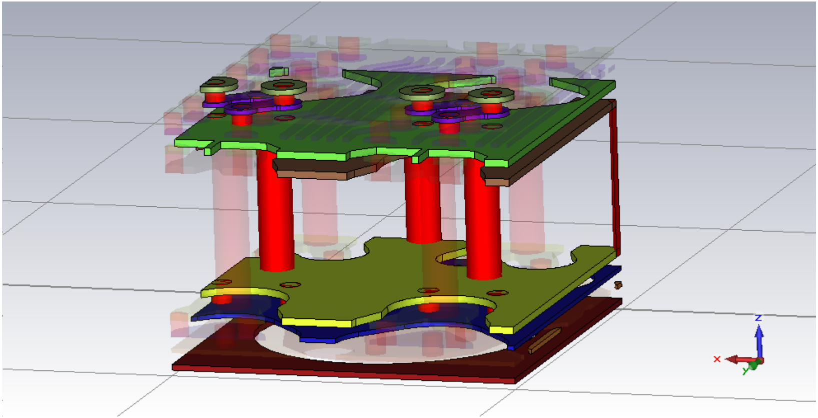
$O(10^9)$ unknowns in full system.

Find Reduced Basis on each Domain



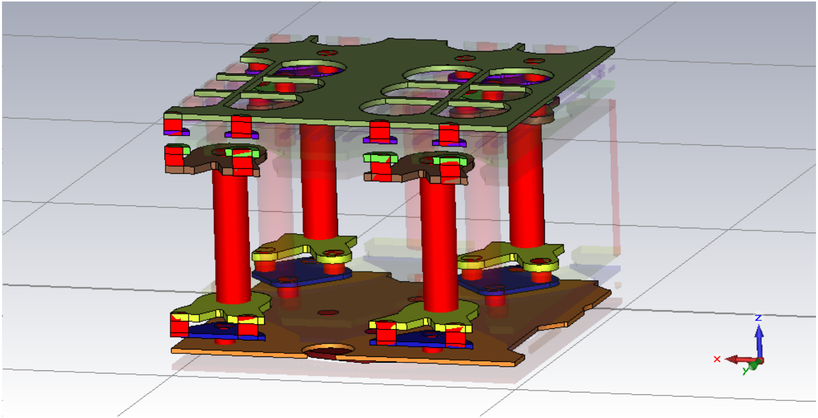
Example Domain

Find Reduced Basis on each Domain



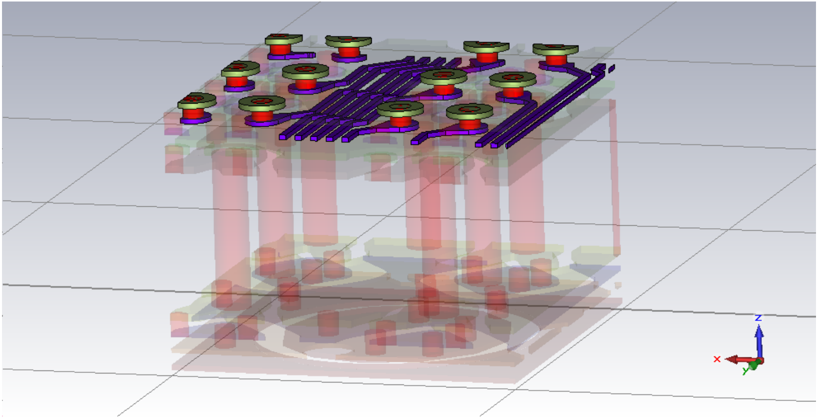
Example Domain, GND

Find Reduced Basis on each Domain



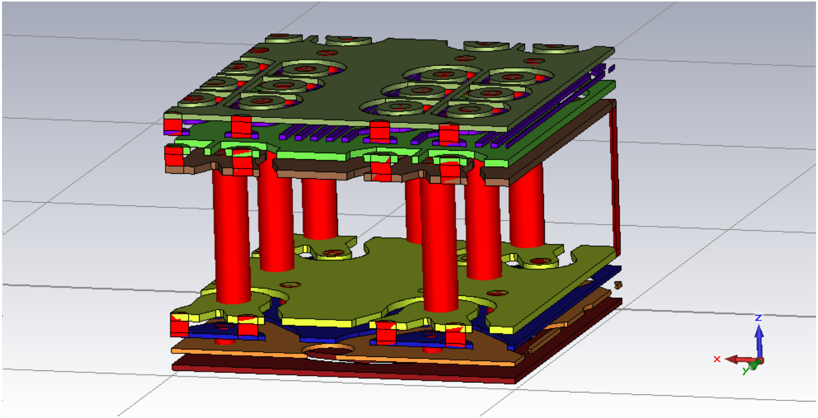
Example Domain, Power

Find Reduced Basis on each Domain



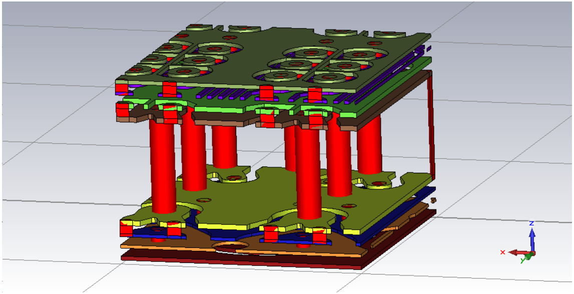
Example Domain, Signals

Find Reduced Basis on each Domain

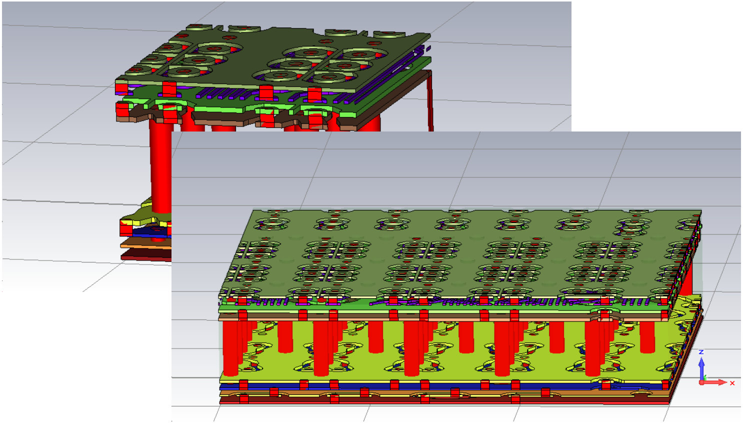


Example Domain

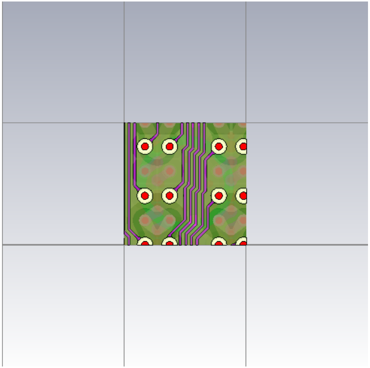
Example Domain With Oversampling



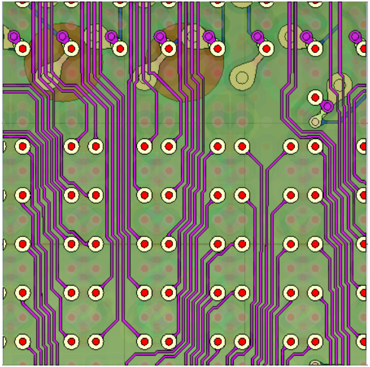
Example Domain With Oversampling



Example Domain With Oversampling, from Top

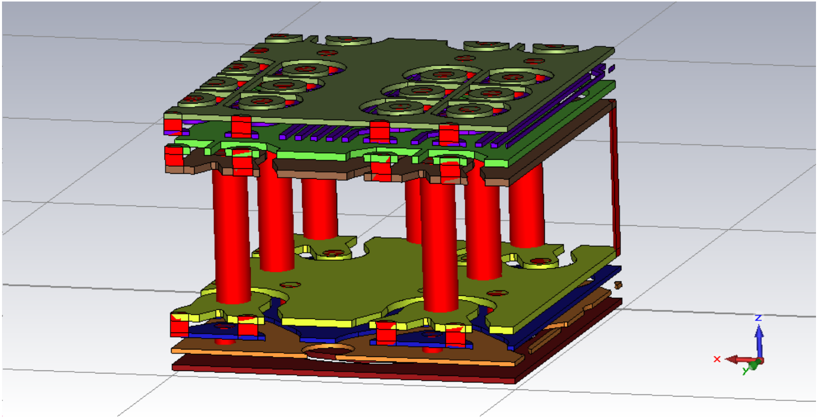


Domain



Domain with Oversampling

Find Reduced Basis on each Domain



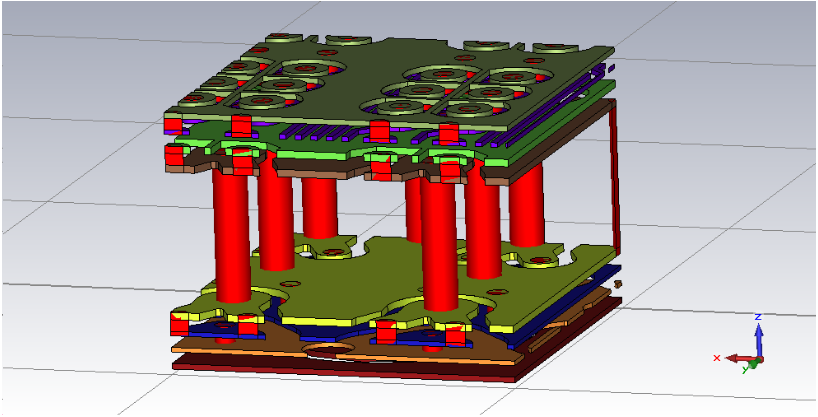
Example Domain

Following SCRBE by Patera et al.

- We need a reduced basis at the domain interfaces.
- These interface basis functions are extended to the volume.
- Maybe we add bubble functions.

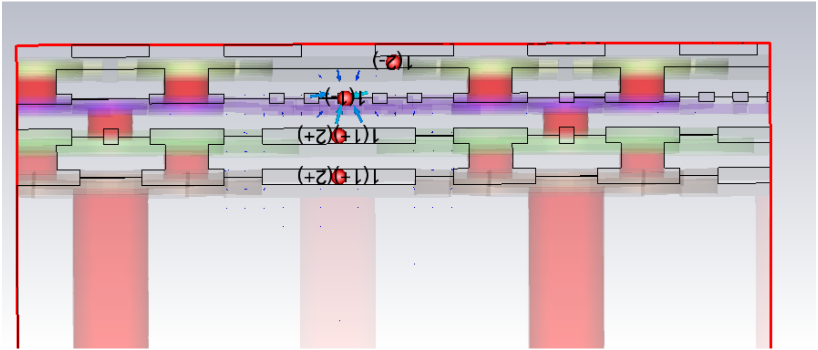
- How to create the reduced basis for the interfaces?

Interface Basis Functions



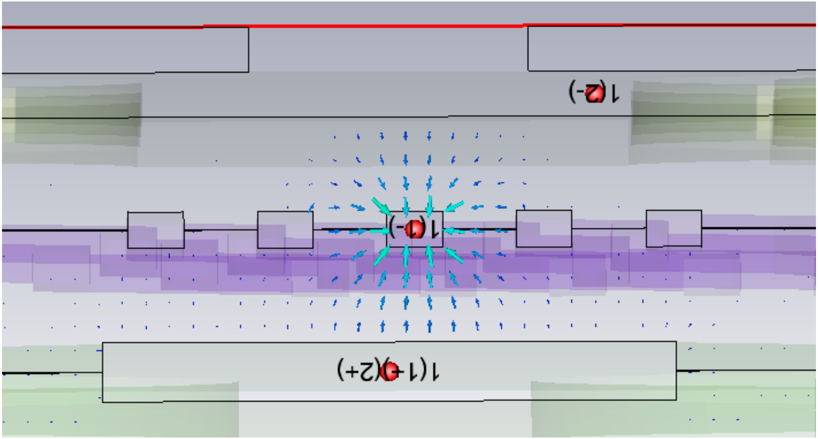
Imagine Signal through interface

Interface Basis Functions



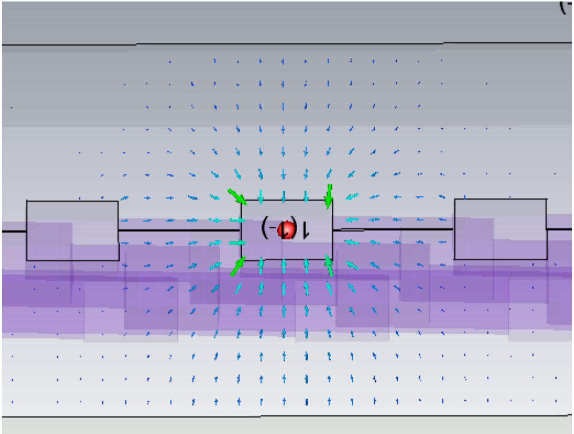
Interface function 1 (not real data)

Interface Basis Functions



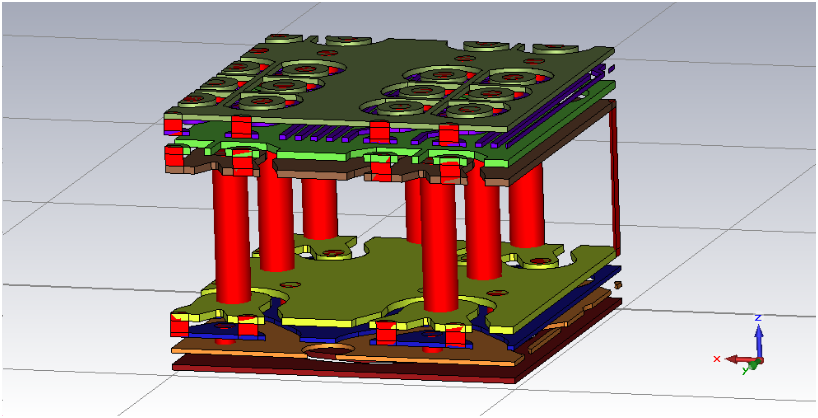
Interface function 1, zoomed (not real data)

Interface Basis Functions



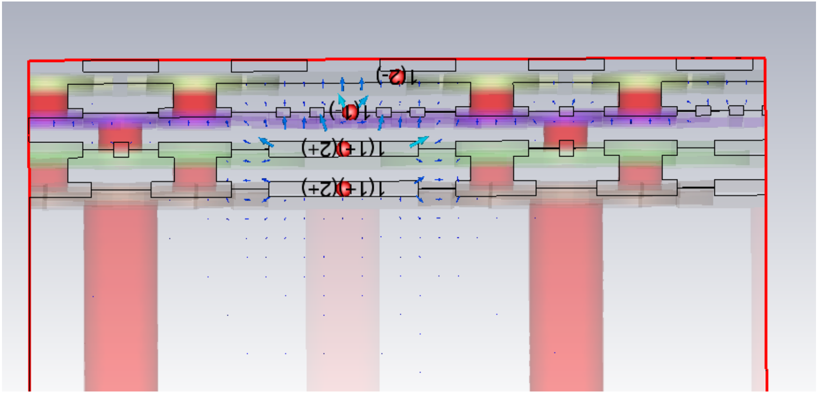
Interface function 1, zoomed even more (not real data)

Interface Basis Functions



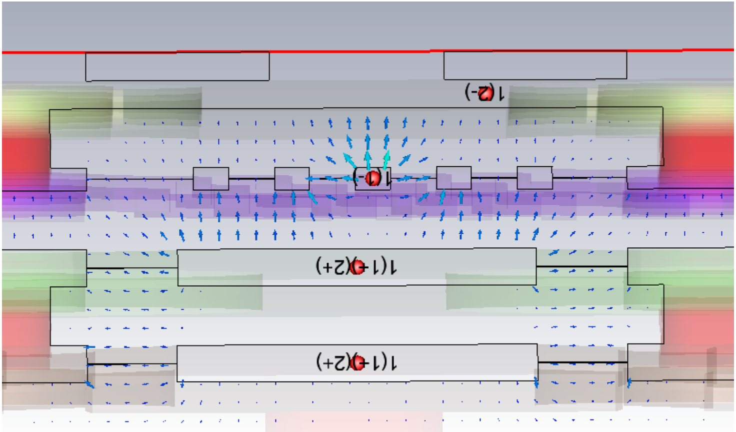
Imagine Potential between plates

Interface Basis Functions



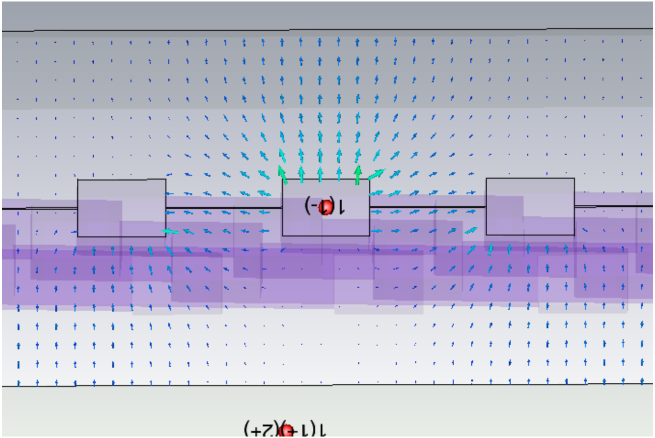
Interface function 2 (not real data)

Interface Basis Functions



Interface function 2, zoomed (not real data)

Interface Basis Functions

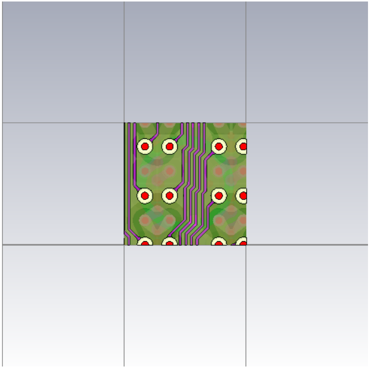


Interface function 2, zoomed even more (not real data)

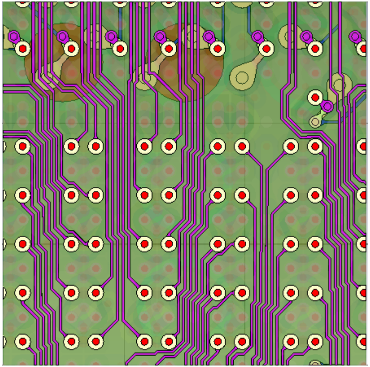
Inspired by GMsFEM by Efendiev et al.[2, 4, 3, 1]

- Take interface in question plus an oversampling.
- Apply all possible boundary conditions.
- Create a basis which can represent (up to ε) the solutions obtained at the interface.

Interface plus Oversampling

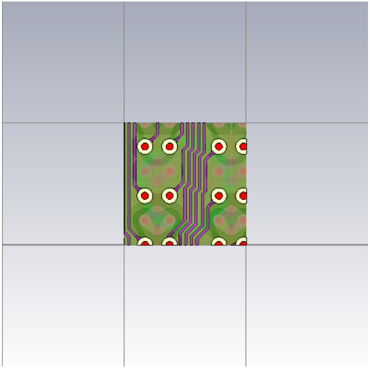


Domain

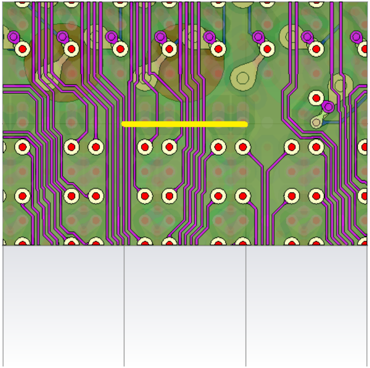


Domain with Oversampling

Interface plus Oversampling



Domain



Interface with Oversampling

Summary

- To achieve *local modifyability* and *scalability* localized interfaces bases and localized volume bases should be created in a completely localized and communication-free algorithm.
- *Interface bases* should be created by taking each interface with an oversampling, applying all possible boundary conditions and constructing an interface basis based on the solutions.
- *Volume bases* should be created by extending interface functions to the volume.

References

Eric T. Chung, Yalchin Efendiev, and Wing Tat Leung.
Generalized multiscale finite element methods for wave propagation in heterogeneous media.
July 2013.

Yalchin Efendiev, Juan Galvis, and Thomas Y. Hou.
Generalized multiscale finite element methods (gmsfem).
January 2013.

Yalchin Efendiev, Juan Galvis, Guanglian Li, and Michael Presho.
Generalized multiscale finite element methods. nonlinear elliptic equations.
April 2013.

Yalchin Efendiev, Juan Galvis, Guanglian Li, and Michael Presho.
Generalized multiscale finite element methods. oversampling strategies.
April 2013.