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Localized Reduced Basis Methods for Time Harmonic Maxwell's Equations

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Abstract

Localized model order reduction methods have attracted significant attention during the last years. They have favorable parallelization properties and promise to perform well on cloud architectures, which become more and more commonplace. We introduced ArbiLoMod [1], a localized reduced basis method targeted at the important use case of changing problem definition, wherein the changes are of local nature. This is a common situation in simulation software used by engineers optimizing a CAD model.
An especially interesting application is the simulation of electromagnetic fields in printed circuit boards, which is necessary to design high frequency electronics. The simulation of the electromagnetic fields can be done by solving the time-harmonic Maxwell's equations, which results in a parameterized, inf-sup stable problem which has to be solved for many parameters. In this multi-query setting, the reduced basis method can perform well. Experiments have shown two dimensional time-harmonic Maxwell's to be amenable to localized model reduction [2].
However, Galerkin projection of an inf-sup stable problem is not guaranteed to be stable. Existing stabilization methods for the reduced basis method involve global computations and are thus not applicable in a localized setting. Replacing the Galerkin projection with the minimization of a localized a posteriori error estimator provides a stable reduction for inf-sup stable projects which retains all the advantageous properties of localized model order reduction. It allows for an offline-online decomposition and requires no global computations in the unreduced space.

Modelling

Maxwell's equations:

$$\begin{aligned}\nabla \times E &= -\frac{\partial}{\partial t} B\\ \nabla \cdot D &= \rho\\ \nabla \times H &= \partial_t D + j\\ \nabla \cdot B &= 0\end{aligned}$$

Time harmonic Maxwell's equations:

$$\nabla \times \frac{1}{\mu} \nabla \times E - \omega^2 \epsilon E = -i\omega j$$

with angular frequency ω.

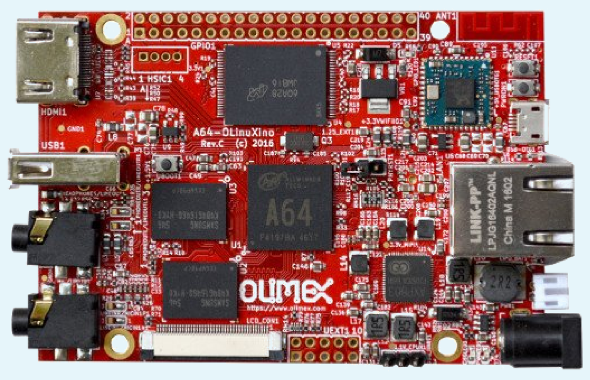
Weak formulation in $H(\mathrm{curl})$:

$$\begin{aligned}a(u,v;\omega) &= \frac{1}{\mu}\int_\Omega (\nabla \times u)\cdot(\nabla \times v)\,\mathrm{d}x - \omega^2 \epsilon \int_\Omega u \cdot v\,\mathrm{d}x\\ f(v;\omega) &= -i\omega \int_\Omega j \cdot v\,\mathrm{d}x\end{aligned}$$

Notation: | E: electric field | D: electric flux | H: magnetic field | B: magnetic flux | ρ: charge density | j: current density | ϵ: electric permittivity | μ: magnetic permeability |

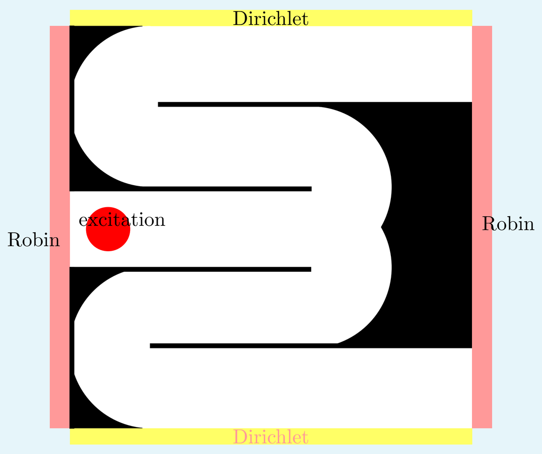
Target Problem

Signal integrity analysis in high speed PCBs (Olimex OLinuXino A64 as example)

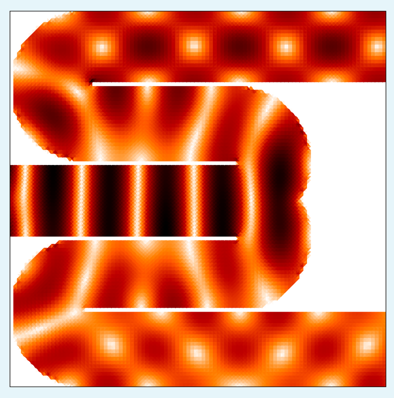


- 3D simulation
- Hexahedral mesh, ≈ 21 million meshcells
- First order Nédélec ansatz functions, ≈ 65 million degrees of freedom
- Domain decomposition with 1120 domains

Test Problem



- 2D simulation
- Triangular mesh
- First order Nédélec ansatz functions, 60.200 degrees of freedom
- Domain decomposition with 81 domains



Localized a Posteriori Error Estimator

The error estimator Δ is defined as a constant times the Euclidean norm of the local dual space norms of the residual [1].

With

$$\Delta(\hat{u}) := \frac{1}{\beta} c_{\mathrm{pu}} \left(\sum_{i=1}^{N_D} \|R(\hat{u})\|^2_{V_i'} \right)^{\frac{1}{2}}.$$

it holds

$$\|u - \hat{u}\|_V \leq \Delta(\hat{u}).$$

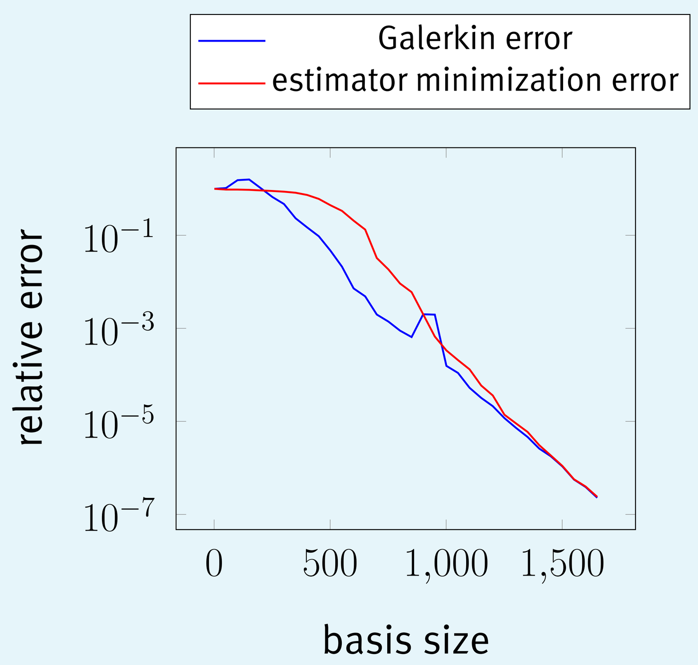
- β: inf-sup constant
- c_{pu}: stability constant associated with partition of unity
- N_D: number of domains
- $R(\hat{u})$: residual of $\hat{u}$ in the dual space
- V_i: local space on domain i

Estimator Minimization

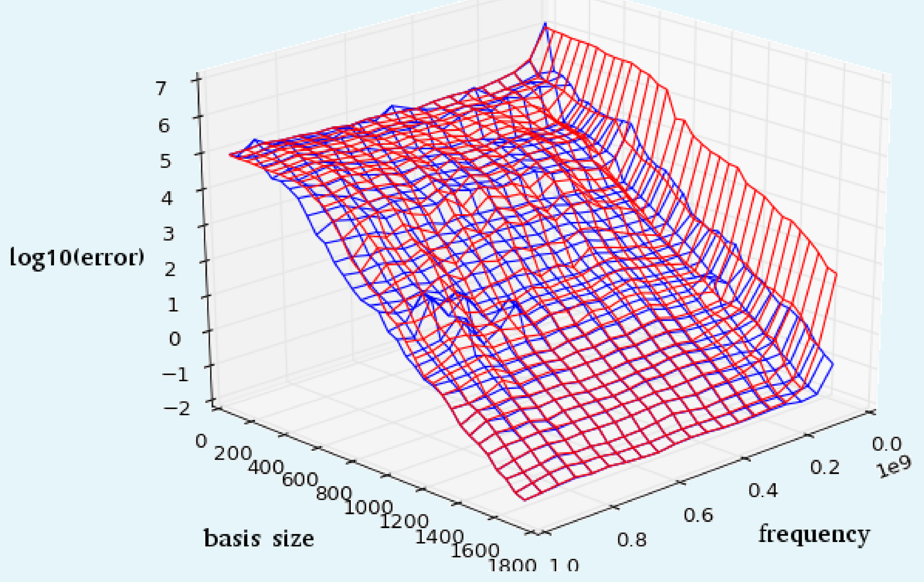
- The full solution u is defined to be the solution of find $u \in V$ s.t.
$$a(u,v;\omega) = f(v;\omega) \qquad \forall v \in V.$$
- The reduced solution $\tilde{u}$ is defined to be the solution of find $\tilde{u} \in \tilde{V}$ s.t.
$$a(\tilde{u},\tilde{v};\omega) = f(\tilde{v};\omega) \qquad \forall \tilde{v} \in \tilde{V}.$$
- The estimator minimization solution $\hat{u}$ is defined by
$$\hat{u} := \operatorname*{argmin}_{\varphi \in \tilde{V}} \Delta(\varphi).$$

Numerical Results

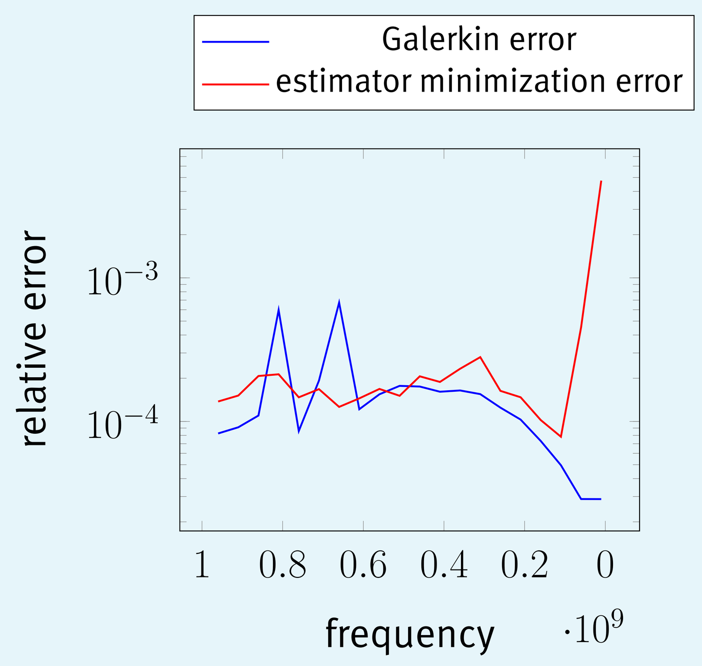
For 860MHz:



Blue: Galerkin error $\|u - \tilde{u}\|$
Red: estimator minimization error $\|u - \hat{u}\|$



For basis size 1051:



Observations

The Galerkin projection of the inf-sup stable problem suffers from instabilities. For some basis sizes, the error peaks. The reduction by minimization of the localized a posteriori error estimator, in contrast, does not suffer from instabilites. However, in most cases, the Galerkin projection obtains a better solution than the error estimator minimization procedure.
The choosen discretization approach suffers from a well known low frequency instability. For frequencies approaching zero, the inf-sup constant gets smaller. This is reflected in the error of the estimator minimization solution: Its error goes up as the inf-sup constant goes down. This effect is much less strong for the Galerkin projection: Its error stays approximately constant, even though the inf-sup constant gets smaller.

References

References

[1] A. Buhr, C. Engwer, M. Ohlberger, and S. Rave. ArbiLoMod, a Simulation Technique Designed for Arbitrary Local Modifications. *SIAM J. Sci. Comput.*, 39(4):A1435–A1465, 2017.

[2] A. Buhr, C. Engwer, M. Ohlberger, and S. Rave. *ArbiLoMod: Local Solution Spaces by Random Training in Electrodynamics*, pages 137–148. Springer International Publishing, Cham, 2017.