Westfälische Wilhelms-Universität Münster

Applied Mathematics Münster

Interactive Simulations Using Localized Reduced Basis Methods

MATHMOD 2015 - Vienna

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WWU Münster

Andreas Buhr

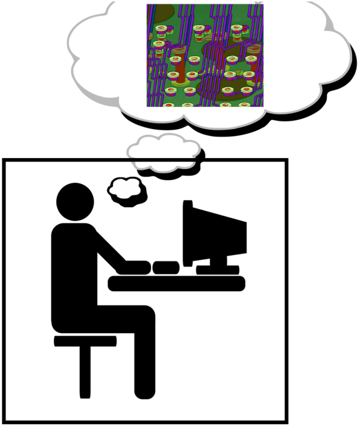
Mario Ohlberger
Stephan Rave

February 19, 2015

Agenda

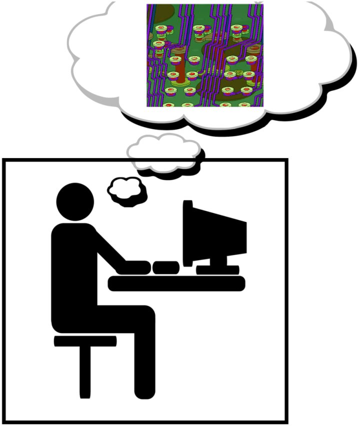
1. Problem
2. Implemented Toolbox
3. Proposed Method
4. Example

Non-Parametric Geometry Changes



Envision engineer working on design

Non-Parametric Geometry Changes



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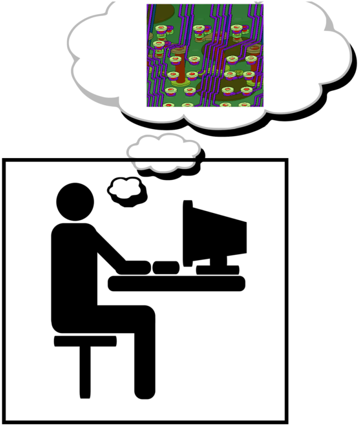
Multi-query setting:
sequence of geometries

Non-Parametric Geometry Changes



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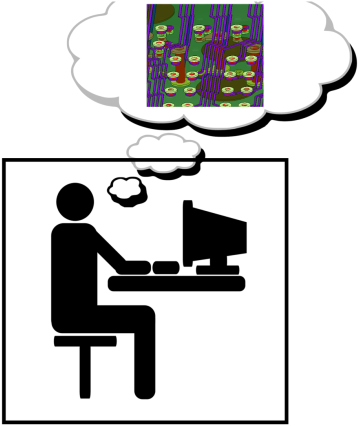
Multi-query setting:
sequence of geometries

Properties of changes:

1. very localized

Non-Parametric Geometry Changes



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Multi-query setting:
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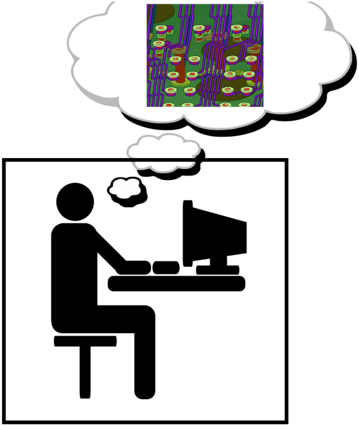
Properties of changes:

1. very localized
2. unforeseen

Non-Parametric Geometry Changes



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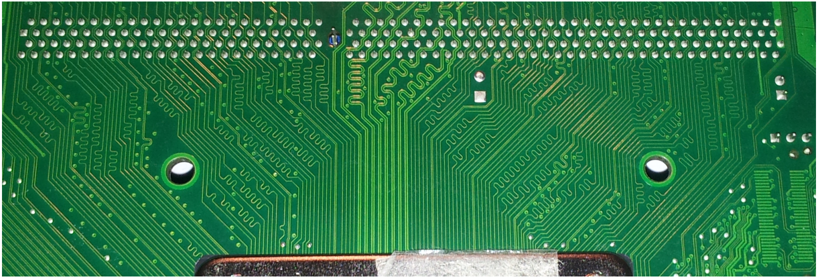
Multi-query setting:
sequence of geometries

Properties of changes:

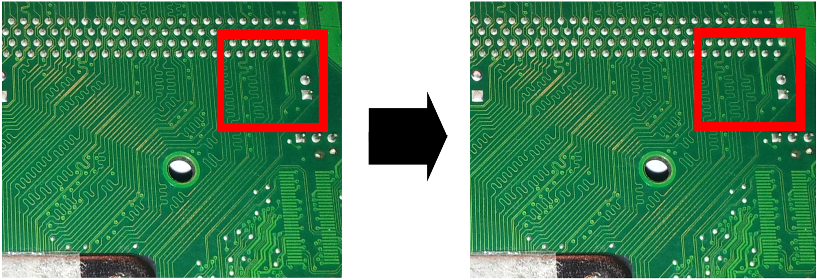
1. very localized
2. unforeseen

Cluster often available.
Cloud always available.

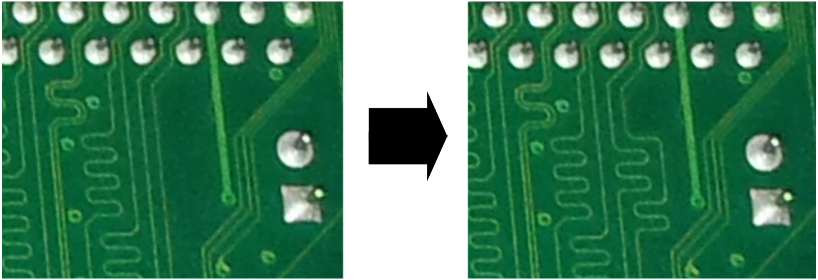
Example



Example



Example



Proposal

- Modify the Reduced Basis Method (RBM) to use basis functions with local support.
- After geometry change, reuse basis functions which do not have support in changed area.

Standard Reduced Basis

Problem

Find u_μ in V such that

$$a_\mu(u_\mu, v) = f_\mu(v) \qquad \forall\, v \in V$$

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Subspace Construction

Construct subspace $\widetilde{V} \subset V$ with $\dim(\widetilde{V}) \ll \dim(V)$,

$$\widetilde{V} := \operatorname{span}\{u_{\mu_1}, u_{\mu_2}, u_{\mu_3}, \ldots\}$$

Standard Reduced Basis

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Subspace Construction - To Be Changed

Construct subspace $\widetilde{V} \subset V$ with $\dim(\widetilde{V}) \ll \dim(V)$,

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Localized Reduced Basis

Reduced Space is spanned by solutions to localized problems.

Implemented Toolbox to Localize Problems

- Nonoverlapping Domain Decomposition:

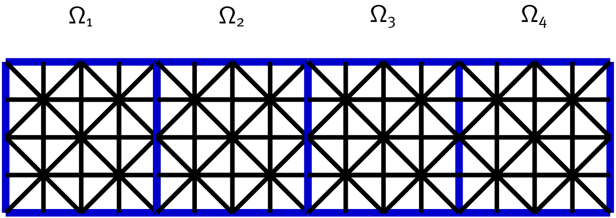
$$\Omega = \bigcup_i \Omega_i \qquad \Omega_i \cap \Omega_j = \emptyset \quad i \neq j$$

- Fine mesh resolving DD
- Finite Element ansatz space
- Grouping of FE ansatz functions by their support

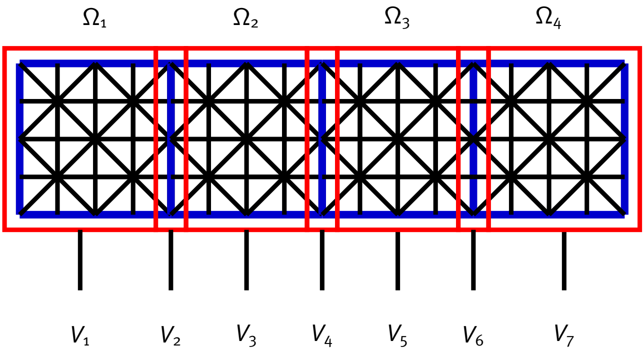
$$V = \bigoplus_k V_k \qquad V_k \subset V$$

- Joining subspaces V_k

Example for Space Decomposition

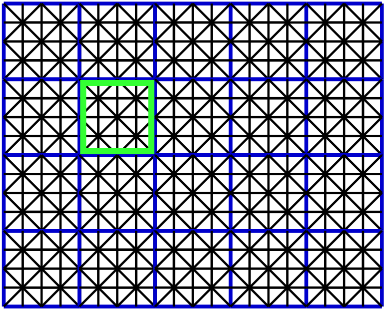


Example for Space Decomposition

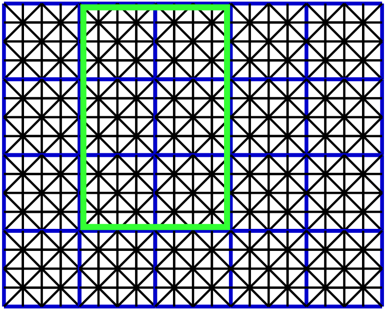


Example for Joined Subspaces

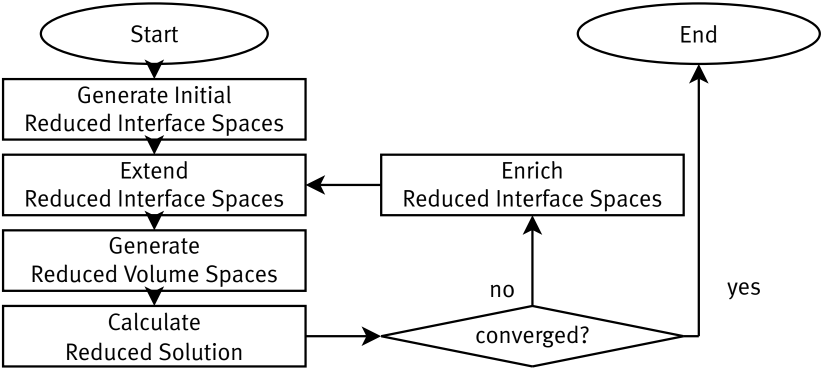
solving in one domain



solving in a patch of six domains



ArbiLoMod Overview

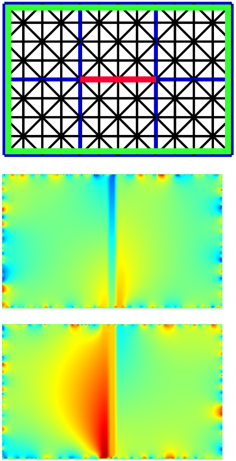


1. Generate Initial Interface Spaces

Adaption of “Pairwise Training” from PR-SCRBE[4],
Sixpack-Training:

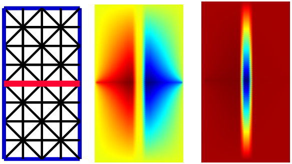
On patch around interface:

- solve for random boundary conditions
- restrict solutions to interface
- create POD of restricted solutions
- use largest POD modes



2. Extend Interface Spaces

Solve on touching domains with interface spaces as boundary values



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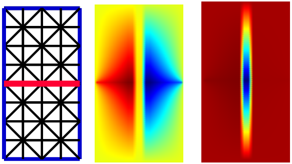
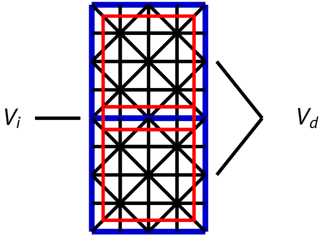




Extension Operator

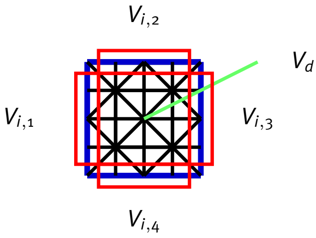
For fixed parameter μ: $E : V_i \to V_i \oplus V_d$

$$\psi \mapsto \psi + \varphi$$

$$a_\mu(\psi + \varphi, v) = 0 \quad \forall v \in V_d$$

3. Generate Volume Spaces

For all interface functions:
For all parameter values in training set:
Create volume basis which can approximate solution.



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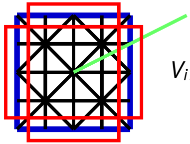

For all interface functions:
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Volume Basis

With parameter training set Ξ:

$$V_{IE} := E(\widetilde{V}_{i,1}) \oplus E(\widetilde{V}_{i,2}) \oplus E(\widetilde{V}_{i,3}) \oplus E(\widetilde{V}_{i,4})$$

$$M_1 := \{\varphi \in V_d \mid a_\mu(\psi + \varphi, v) = 0 \quad \forall v \in V_d, \mu \in \Xi, \psi \in V_{IE}\}$$

$$M_2 := \{\varphi \in V_d \mid a_\mu(\varphi, v) = f_\mu(v) \quad \forall v \in V_d, \mu \in \Xi\}$$

$$\widetilde{V}_d := \text{greedy}(M_1 \cup M_2)$$

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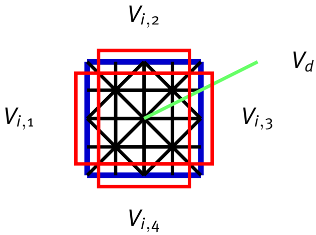

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$$\widetilde{V}_d := \text{greedy}(M_1 \cup M_2)$$

(left out node spaces)

4. Calculate Reduced Solution

Reduced Problem

Find $\widetilde{u}_\mu$ in $\widetilde{V}$ such that

$$a_\mu(\widetilde{u}_\mu, \widetilde{v}) = f_\mu(\widetilde{v}) \qquad \forall\, \widetilde{v} \in \widetilde{V}$$

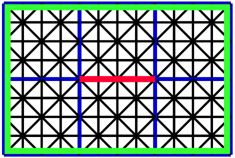
Reduced Space $\widetilde{V}$

$$\widetilde{V} := \bigoplus_i E(\widetilde{V}_{I,i}) \oplus \bigoplus_i \widetilde{V}_{D,i}$$

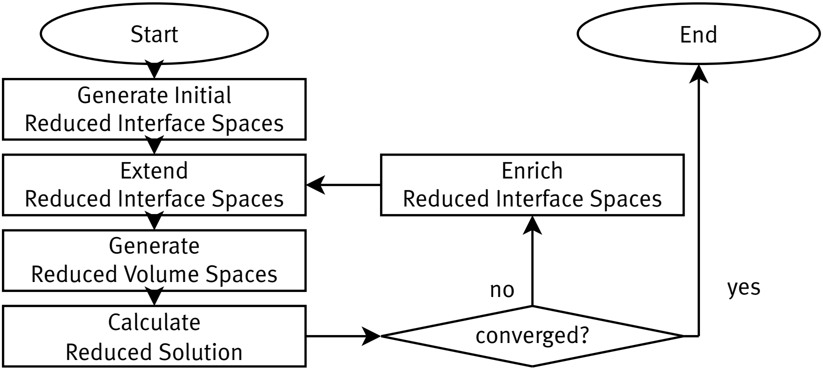
5. Enrich Interface Spaces

On patch around interface:

- Use last solution as boundary values
- Solve on patch
- restrict solution to interface
- enrich interface space

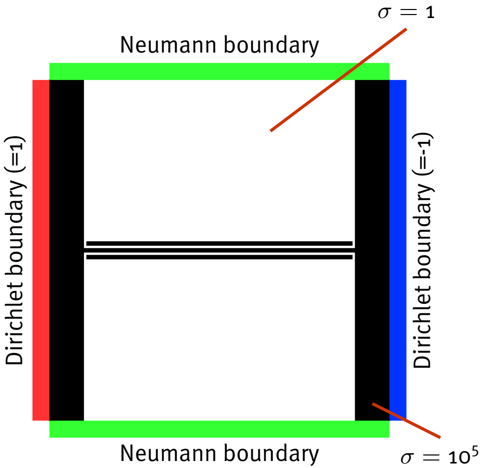


ArbiLoMod Overview



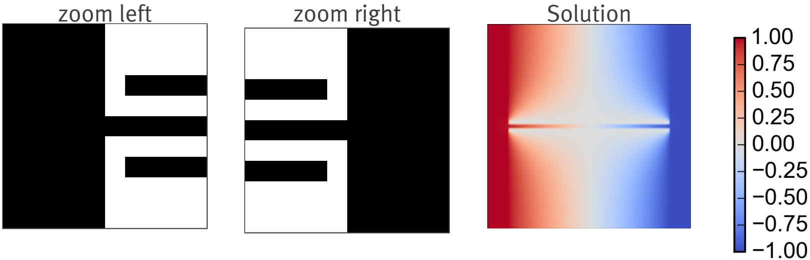
Numerical Example

Heat conduction
$-\nabla \cdot (\sigma \nabla u) = 0$



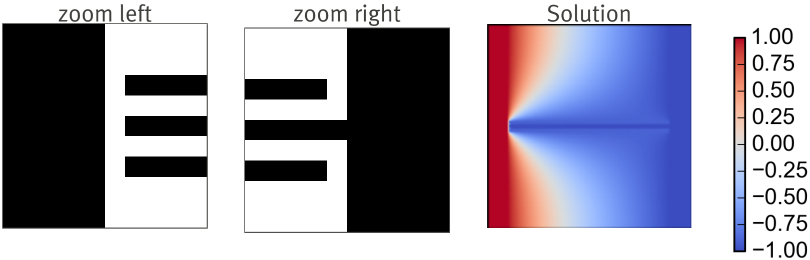
Sequence of Geometries

Geometry 1:



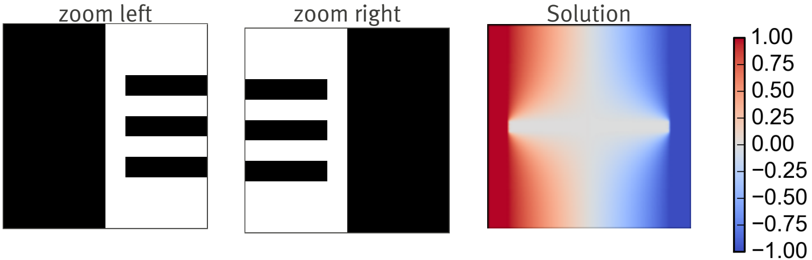
Sequence of Geometries

Geometry 2:



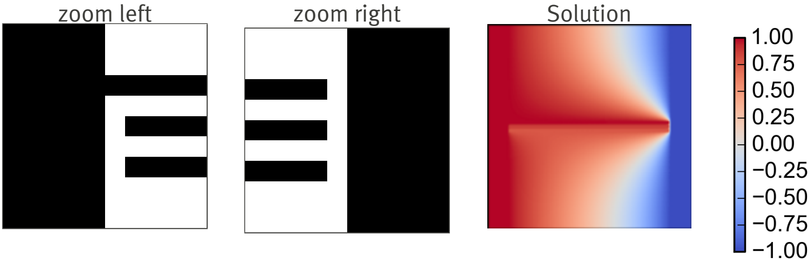
Sequence of Geometries

Geometry 3:



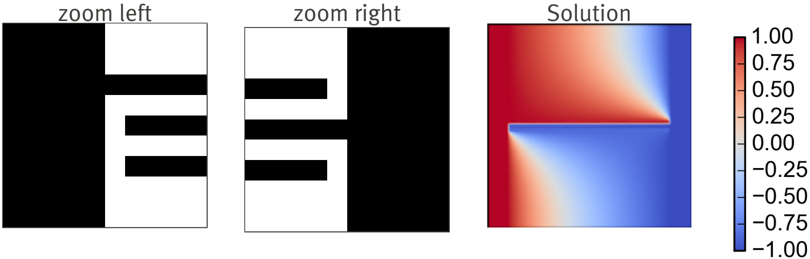
Sequence of Geometries

Geometry 4:



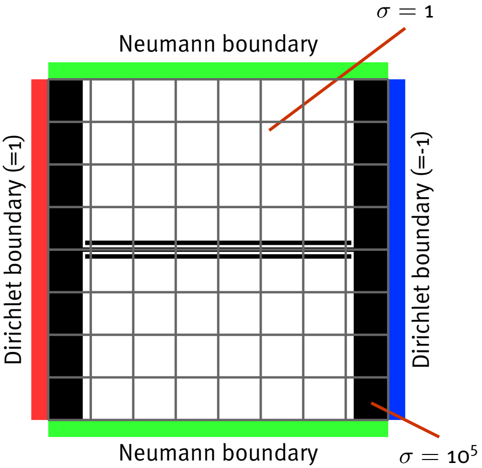
Sequence of Geometries

Geometry 5:

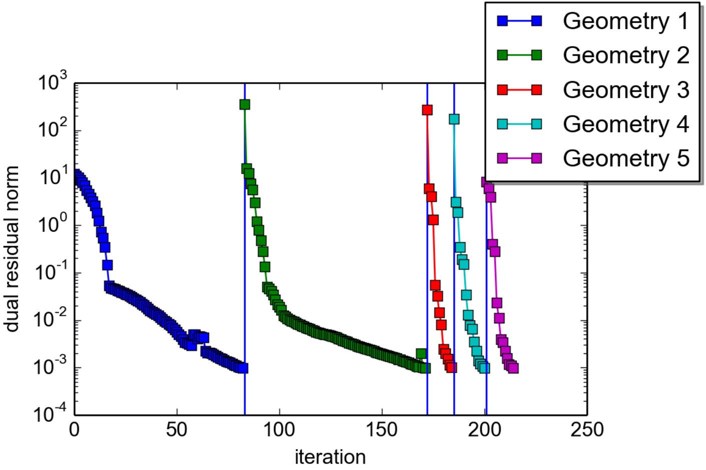


Numerical Example

- Heat conduction
  $-\nabla \cdot (\sigma \nabla u) = 0$
- 8x8 domain decomposition
- 80.401 dofs in full model



Convergence



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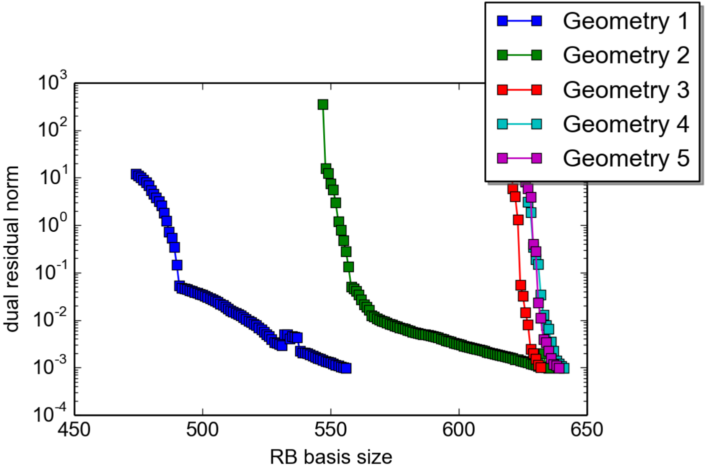
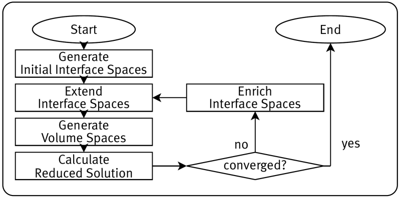


Summary



ArbiLoMod

- ... handles arbitrary local modifications.
- ... intended for interactive use.
- ... is based on the Reduced Basis Method.
- ... has localized and adaptive generation of basis vectors.

References

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The localized reduced basis multiscale method.
Proceedings of Algoritmy 2012, Conference on Scientific Computing, Vysoke Tatry, Podbanske, September 9-14, 2012, pages 393–403, 2012.

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Master's thesis, TU Darmstadt, 2009.

Yalchin Efendiev, Juan Galvis, and Thomas Y. Hou.
Generalized multiscale finite element methods (gmsfem).
January 2013.

Jens L Eftang and Anthony T Patera.
A port-reduced static condensation reduced basis element method for large component-synthesized structures: approximation and a posteriori error estimation.
Advanced Modeling and Simulation in Engineering Sciences, 1(1):3, 2014.

Acknowledgements

Many thanks to...

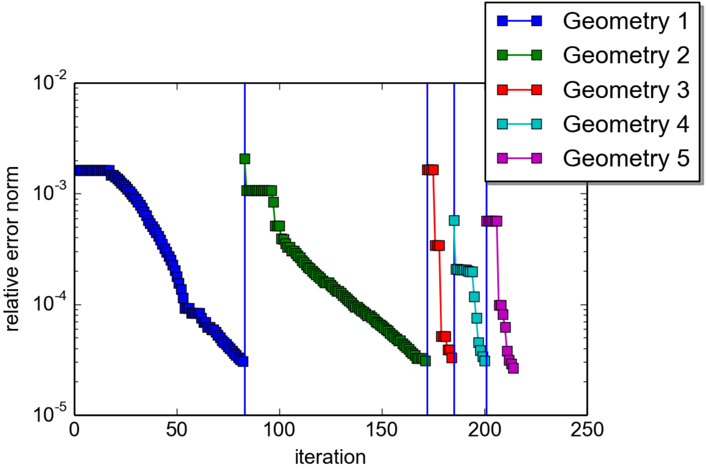
CST - Computer Simulation Technology AG[1]

... for sponsoring my research.

[1] www.cst.com

Backup Slides

Convergence



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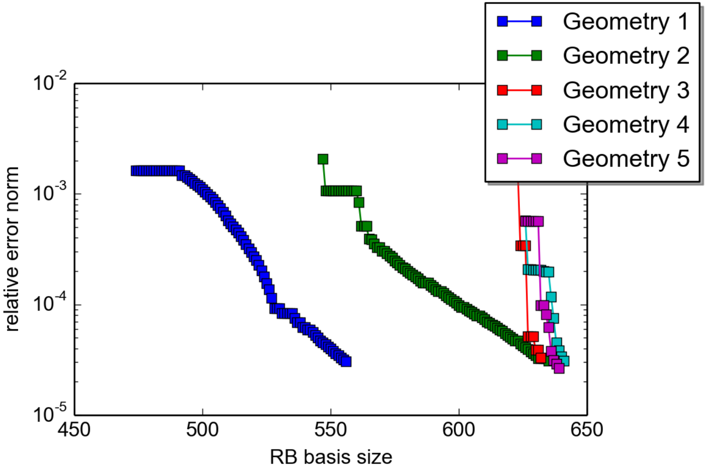

Sparse Matrix